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# Comments on Open-String Orbifolds with a Non-Vanishing $B_{ab}$

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## Abstract

We study the effect of a non-vanishing flux for the NS-NS antisymmetric tensor in open-string orbifolds. As in toroidal models, the total dimension of the Chan-Paton gauge group is reduced proportionally to the rank of  $B_{ab}$ , both on D9 and on D5-branes, while the Möbius amplitude involves some signs that, in the  $Z_2$  orbifold, allow one to connect continuously  $U(n)$  groups to  $Sp(n) \otimes Sp(n)$  groups on each set of D-branes. In this case, non-universal couplings between twisted scalars and gauge vectors arise, as demanded by the generalised Green-Schwarz mechanism. We also comment on the role of the NS-NS antisymmetric tensor in a recently proposed type I scenario, where supersymmetry is broken on the D-branes, while it is preserved in the bulk.

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## 1. Introduction

Since the initial proposal to identify open strings as parameter space orbifolds of closed oriented strings [1], much progress has been made in our understanding of the rules that determine the construction of open-string vacua [2,3,4]. In particular, the role played by modular invariance for oriented closed strings is played by the tadpole conditions and by a consistent interpretation of the various amplitudes. Indeed, due to lack of modular invariance, the Klein bottle, annulus and Möbius amplitudes develop UV divergences that can be related to tadpoles for unphysical Ramond-Ramond (R-R) fields [5]. Tadpole conditions, then, ensure that the dangerous UV divergences are cured and, as result, the total flux of the R-R fields vanishes consistently [6].

A consistent interpretation means that all the amplitudes must provide a coherent pattern of (space-time) massless and massive excitations in the direct vacuum-channel, consistent with spin-statistics and, after  $S$  or  $P$  modular transformations, should describe a consistent tree-level dynamics of (projected) closed string states bouncing between holes and/or crosscaps. A precise set of rules has been classified in [2,4], and can be formally summarised as follows. Starting from a torus amplitude

$$\mathcal{T} = \sum_{i,j} \chi_i N_{ij} \bar{\chi}_j$$

that encodes the spectrum of the parent closed string theory, one effectively realizes the world-sheet parity projection  $\Omega$  by adding the Klein bottle amplitude

$$\mathcal{K} = \frac{1}{2} \sum_i N_{ii} \chi_i .$$

The restriction to the diagonal combinations  $N_{ii}$  guarantees a consistent particle interpretation, while an  $S$ -modular transformation shows that the transverse-channel amplitude

$$\tilde{\mathcal{K}} = \sum_i \Gamma_i^2 \chi_i$$

involves coefficients that are perfect squares and represent one-point functions of closed fields in the presence of a crosscap.

Open unoriented strings correspond to the twisted sector of the world-sheet orbifold and their relevant contributions to the one-loop partition function are encoded in the annulus amplitudes

$$\mathcal{A} = \frac{1}{2} \sum_{i,a,b} A_{ab}^i \chi_i n^a n^b , \quad \tilde{\mathcal{A}} = \sum_i B_i^2 \chi_i ,$$

and in the Möbius amplitudes ( $M_a^i = A_{aa}^i \bmod 2$ )

$$\mathcal{M} = \frac{1}{2} \sum_{i,a} M_a^i \hat{\chi}_i n^a, \quad \tilde{\mathcal{M}} = 2 \sum_i \Gamma_i B_i \hat{\chi}_i.$$

Here  $\tilde{\mathcal{A}}$  respects the rule of perfect squares with, now,  $B_i$  the one-point function of closed states in the presence of a disk. Then,  $\tilde{\mathcal{M}}$  is the geometric mean of  $\tilde{\mathcal{K}}$  and  $\tilde{\mathcal{A}}$ , as expected since the Möbius strip has a single boundary and a single crosscap. In the direct channel the restriction to  $M_a^i = A_{aa}^i \bmod 2$  ensures a consistent particle interpretation in the open sector where the  $n^a$ 's represent Chan-Paton multiplicities.

In this paper we shall apply these general rules to the case of orbifold compactifications in the presence of a quantised NS-NS antisymmetric tensor background<sup>¶</sup>. In toroidal compactifications of the open strings [8] it was shown that, although the corresponding modes are projected out by  $\Omega$ , a *quantised*  $B_{ab}$  background is still consistent with world-sheet parity. The main results in [8] were that the size the Chan-Paton gauge group is reduced in way that depends on the rank of  $B_{ab}$ , and that SO and Sp gauge groups may be connected by (continuous) Wilson lines and sign ambiguities that enforce a proper normalisation of the Möbius amplitude. These results explained the key features of the gauge groups found in the type-I compactifications studied in [2,9]. Our main purpose here is to construct the full amplitudes (at a generic, irrational, point in moduli space [3,10]), following the rules we have previously summarised, and then to recover some geometric insights [11] added to the new phenomena discovered in [9] from the constraints (and freedoms) imposed on the amplitudes. We shall thus give an interpretation to the appearance of varying numbers of tensor multiplets in the closed sector and we shall be able to identify the signs present in  $\mathcal{M}$  with discrete Wilson lines.

This paper is organised as follows. In section 2 we review the toroidal compactifications of the type I superstring in the presence of a non-vanishing  $B_{ab}$ . In section 3 we generalise this construction, discussing in detail the structure of the various amplitudes for the  $T^4/Z_2$  orbifold. In section 4 we give a physical interpretation of the results presented in section 3, solving the tadpole conditions and determining the massless excitations for the various models. We also establish a connection between discrete Wilson lines and the signs in  $\mathcal{M}$ , and, finally, we comment on the emergence of non-universal couplings between twisted

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<sup>¶</sup> For a previous, partial, description of orbifolds with a non-vanishing  $B_{ab}$ , without a complete construction of the partition functions, see [7].

scalars and gauge vectors, consistently with the generalised Green-Schwarz mechanism. In section 5 we generalise the construction of the open descendants of the  $Z$  orbifold [12], thus emphasising the role played by the signs in the Möbius amplitude. In section 6 we apply our results to a recently proposed type I scenario [13], where supersymmetry is broken on branes but remains unbroken in the bulk. Section 7 contains our conclusions.

## 2. Review of generalised toroidal compactifications

Let us review the compactification of open strings on a  $d$ -dimensional torus, studied for the first time in [8]. On a generic  $d$ -dimensional lattice, the left and right momenta of the parent closed theory can be expressed as [14]

$$\begin{aligned} p_a &= m_a + \frac{1}{\alpha'}(g_{ab} - B_{ab})n^b, \\ \tilde{p}_a &= m_a - \frac{1}{\alpha'}(g_{ab} + B_{ab})n^b, \end{aligned} \tag{2.1}$$

where the metric  $g_{ab}$  and the NS-NS antisymmetric tensor  $B_{ab}$  describe the size and the shape of the internal torus. It is evident that, at generic points in moduli space, the parent theory is no more left-right symmetric. Nonetheless, world-sheet symmetry still holds if one restricts the attention to particular tori, for which effectively  $p_a = \tilde{p}_a$ . As a result, the combination  $\frac{2}{\alpha'}B_{ab}$  need be an integer, *i.e.* the antisymmetric tensor is quantised in appropriate units. This is consistent with the fact that, in the open descendants, the fluctuations of the NS-NS two-tensor are projected out of the spectrum, so that only the moduli of the internal metric can be used to deform the lattice.

Once the left-right symmetry of the parent closed string is restored, one can proceed to construct the open descendants, starting, as usual, with the Klein bottle amplitudes

$$\mathcal{K}^{(\text{tor})} = \frac{1}{2}(V_8 - S_8)(q^2) \sum_m \frac{q^{\frac{\alpha'}{2}m^T g^{-1}m}}{\eta^d(q^2)}$$

and

$$\tilde{\mathcal{K}}^{(\text{tor})} = \frac{2^5}{2} \sqrt{\det(g/\alpha')} (V_8 - S_8)(i\ell) \sum_n \frac{(e^{-2\pi\ell})^{\frac{1}{\alpha'}n^T g n}}{\eta^d(i\ell)}.$$

Here and in the following, a tilde denotes amplitudes in the transverse channel, and “vertical” and “horizontal” proper times are related by an  $S$  modular transformation for the Klein bottle and annulus amplitudes and by a  $P = T^{\frac{1}{2}}ST^2ST^{\frac{1}{2}}$  transformation for the Möbius amplitude.

The construction of the open-string sector presents some interesting new features. Following [1,2], the transverse-channel annulus amplitude involves only characters that fuse into the identity with their anti-holomorphic partners in the closed-string GSO. In our case, this translates in the restriction to states with  $p_a = -\tilde{p}_a$ . Given the quantisation condition on the  $B$ -field, only the winding states satisfying

$$\frac{2}{\alpha'} B_{ab} n^b = 2m_a \quad (2.2)$$

can contribute to the transverse-channel annulus amplitude. Inserting a projector that effectively realizes (2.2), one gets

$$\tilde{\mathcal{A}}^{(\text{tor})} = \frac{2^{r-d-5}}{2} \sqrt{\det(g/\alpha')} N^2 (V_8 - S_8)(i\ell) \sum_{\epsilon=0,1} \sum_n \frac{(e^{-2\pi\ell})^{\frac{1}{4\alpha'}} n^T g n e^{\frac{2i\pi}{\alpha'} n^T B \epsilon}}{\eta^d(i\ell)}$$

and

$$\mathcal{A}^{(\text{tor})} = \frac{2^{r-d}}{2} N^2 (V_8 - S_8)(\sqrt{q}) \sum_{\epsilon=0,1} \sum_m q^{\frac{\alpha'}{2}(m + \frac{1}{\alpha'} B \epsilon)^T g^{-1}(m + \frac{1}{\alpha'} B \epsilon)} \frac{1}{\eta^d(\sqrt{q})},$$

where  $r = \text{rank}(B)$ , and the normalisation in  $\mathcal{A}$  guarantees that the massless vector has the right multiplicity.

The Möbius amplitudes

$$\begin{aligned} \tilde{\mathcal{M}}^{(\text{tor})} = & -\frac{2 \times 2^{(r-d)/2}}{2} \sqrt{\det(g/\alpha')} N (\hat{V}_8 - \hat{S}_8)(i\ell + \frac{1}{2}) \times \\ & \times \sum_{\epsilon=0,1} \sum_n \frac{(e^{-2\pi\ell})^{\frac{1}{\alpha'}} n^T g n e^{\frac{2i\pi}{\alpha'} n^T B \epsilon} \gamma_\epsilon}{\hat{\eta}^d(i\ell \frac{1}{2})} \end{aligned}$$

and

$$\mathcal{M}^{(\text{tor})} = -\frac{2^{(r-d)/2}}{2} N (\hat{V}_8 - \hat{S}_8)(-\sqrt{q}) \sum_{\epsilon=0,1} \sum_m q^{\frac{\alpha'}{2}(m + \frac{1}{\alpha'} B \epsilon)^T g^{-1}(m + \frac{1}{\alpha'} B \epsilon)} \frac{\gamma_\epsilon}{\hat{\eta}^d(-\sqrt{q})}$$

complete the  $\Omega$ -projection in the open-string sector. As is [2], in order to compensate the fixed real part of the modulus of the doubly-covering torus, we have introduced real “hatted” characters. Notice that the Möbius amplitudes involve crucial signs  $\gamma_\epsilon$  that enforce a correct normalisation of the  $B$ -field projector.

The last step in the construction of open descendants consists in imposing the cancellation of tadpoles of unphysical massless states that flow in the transverse channel. This gives the well known result

$$N = 2^{5-r/2},$$

*i.e.* a non-trivial quantised background for the NS-NS antisymmetric tensor of rank  $r$  reduces the rank of the Chan-Paton gauge group by a factor  $2^{r/2}$ . In geometrical terms, the identification  $B = \frac{1}{2}w_2$ , with  $w_2$  the generalised second Stieffel-Whitney class, suggests to interpret the presence of a quantised  $B_{ab}$  as an obstruction to define a vector structure on the compactification torus [15,16,11].

Before closing this section, let us comment on the role of the signs  $\gamma_\epsilon$  that must be introduced in the Möbius amplitude. To this end, it is convenient to analyse in some detail the case of a two-torus. The metric  $g_{ab}$  and the antisymmetric tensor can be conveniently parametrised as

$$g = \frac{\alpha' Y_2}{X_2} \begin{pmatrix} 1 & X_1 \\ X_1 & |X|^2 \end{pmatrix}, \quad B = \frac{\alpha'}{2} \begin{pmatrix} 0 & Y_1 \\ -Y_1 & 0 \end{pmatrix},$$

where  $X$  and  $Y$  define the complex and Kähler structures, respectively.

The corresponding annulus amplitudes are

$$\tilde{\mathcal{A}} = \frac{2^{-5}}{2} Y_2 N^2 (V_8 - S_8)(i\ell) \sum_{n_1, n_2} \frac{W_{n_1, n_2} [1 + (-1)^{n_1} + (-1)^{n_2} + (-1)^{n_1+n_2}]}{\eta^2(i\ell)},$$

$$\mathcal{A} = \frac{1}{2} N^2 (V_8 - S_8)(\sqrt{q}) \frac{P_{0,0} + P_{0,1} + P_{1,0} + P_{1,1}}{\eta^2(\sqrt{q})},$$

and, in a similar fashion, the Möbius amplitudes are

$$\tilde{\mathcal{M}} = -\frac{2}{2} Y_2 N (\hat{V}_8 - \hat{S}_8)(i\ell + \frac{1}{2}) \sum_{n_1, n_2} \frac{W_{n_1, n_2}}{\hat{\eta}^2(i\ell + \frac{1}{2})}$$

$$\times [\gamma_{0,0} + (-1)^{n_1} \gamma_{0,1} + (-1)^{n_2} \gamma_{1,0} + (-1)^{n_1+n_2} \gamma_{1,1}],$$

$$\mathcal{M} = -\frac{1}{2} N (\hat{V}_8 - \hat{S}_8)(-\sqrt{q}) \frac{[\gamma_{0,0} P_{0,0} + \gamma_{0,1} P_{0,1} + \gamma_{1,0} P_{1,0} + \gamma_{1,1} P_{1,1}]}{\hat{\eta}^2(-\sqrt{q})}.$$

In these expressions we have introduced a compact notation for the winding and momentum sums:

$$W_{n_1, n_2} = (e^{-2\pi\ell})^{\frac{Y_2}{4X_2}} [(n_1 + X_1 n_2)^2 + n_2^2 X_2^2],$$

$$P_{\epsilon_1, \epsilon_2} = \sum_{m_1, m_2} q^{\frac{1}{2X_2 Y_2} [(m_1 + \frac{\epsilon_2}{2} - X_1(m_2 - \frac{\epsilon_1}{2}))^2 + (m_2 - \frac{\epsilon_1}{2})^2 X_2^2]}.$$

Expanding in powers of  $q$ , one can easily find the contribution of each amplitude to the tadpole of the R-R 10-form:

$$\tilde{\mathcal{K}} \sim 2^5,$$

$$\tilde{\mathcal{A}} \sim 2^{-5} \times N^2 \times 4,$$

$$\tilde{\mathcal{M}} \sim -2 \times N \times (\gamma_{0,0} + \gamma_{0,1} + \gamma_{1,0} + \gamma_{1,1}).$$

It is then evident that, in order to solve the tadpole condition, one of the four  $\gamma_\epsilon$  has to equal minus one, while the three others have to equal plus one. Among the four possible choices, only two lead to different results and differ in the sign of  $\gamma_{0,0}$ . If  $\gamma_{0,0} = +1$ , the massless vector in the direct channel belongs to the adjoint of an  $\text{SO}(16)$  gauge group, while if  $\gamma_{0,0} = -1$  it belongs to a symplectic group.

These signs  $\gamma_\epsilon$  are related to the existence of different kinds of orientifold planes with opposite R-R charge [11]. As we shall see in the next sections, a careful construction of the partition functions reveals neatly their role, and shows that they are also responsible for similar “group transitions” in orbifold compactifications.

### 3. The $T^4/Z_2$ orbifold with a non-vanishing $B_{ab}$

Let us now turn to the compactification on the irrational  $T^4/Z_2$  orbifold, in the presence of a non-vanishing background for the NS-NS antisymmetric tensor.

To this end let us introduce the combinations [2]

$$\begin{aligned} Q_O &= V_4 O_4 - C_4 C_4, & Q_S &= O_4 C_4 - S_4 O_4, \\ Q_V &= O_4 V_4 - S_4 S_4, & Q_C &= V_4 S_4 - C_4 V_4, \end{aligned}$$

of level one  $\text{SO}(4)$  characters, that represent the contributions of the world-sheet fermions to the partition function. They are eigenstates of the  $Z_2$  generator, with eigenvalues  $\pm 1$  for  $(Q_O, Q_S)$  and  $(Q_V, Q_C)$ , respectively. Including the contributions of the internal bosons, the partition function for the parent type IIB superstring then reads:

$$\begin{aligned} \mathcal{T} = \frac{1}{2} & \left[ |Q_O + Q_V|^2 \sum_{m,n} \frac{q^{\frac{\alpha'}{4} p g^{-1} p} \bar{q}^{\frac{\alpha'}{4} \tilde{p} g^{-1} \tilde{p}}}{|\eta^4|^2} + |Q_O - Q_V|^2 \left| \frac{\vartheta_3^2 \vartheta_4^2}{\eta^4} \right|^2 + \right. \\ & \left. + |Q_S + Q_C|^2 \left| \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4} \right|^2 + |Q_S - Q_C|^2 \left| \frac{\vartheta_2^2 \vartheta_4^2}{\eta^4} \right|^2 \right], \end{aligned} \quad (3.1)$$

where we have omitted the integration measure and the contributions of non-compact bosonic coordinates. In order to extract the massless spectrum and determine the correct expressions for the direct-channel Klein bottle amplitude and the transverse-channel annulus amplitude, one can extract the contributions at the origin from the lattice sums and rewrite the torus amplitude in terms of “generalised” characters:

$$\begin{aligned} \mathcal{T}_0 \sim & |Q_O \phi_O + Q_V \phi_V|^2 + |Q_O \phi_V + Q_V \phi_O|^2 + \\ & + 16 (|Q_S \phi_S + Q_C \phi_C|^2 + |Q_S \phi_C + Q_C \phi_S|^2), \end{aligned} \quad (3.2)$$



where we have introduced the combinations

$$\begin{aligned}\phi_O &= \frac{1}{2} \left( \frac{1}{\eta^4} + \frac{\vartheta_3^2 \vartheta_4^2}{\eta^4} \right), & \phi_S &= \frac{1}{8} \left( \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4} + \frac{\vartheta_2^2 \vartheta_4^2}{\eta^4} \right), \\ \phi_V &= \frac{1}{2} \left( \frac{1}{\eta^4} - \frac{\vartheta_3^2 \vartheta_4^2}{\eta^4} \right), & \phi_C &= \frac{1}{8} \left( \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4} - \frac{\vartheta_2^2 \vartheta_4^2}{\eta^4} \right).\end{aligned}$$

The numerical factor in (3.2) counts the number of points left fixed by the action of the orbifold generator, and thus gives multiplicities to states in the twisted sector. For a  $Z_2$  orbifold it is equal to 16, as one expects from the Lefschetz theorem [17].

Expanding the “generalised” characters to the leading (massless) order in  $q$ , one gets

$$\begin{aligned}Q_O \phi_O + Q_V \phi_V &\sim V_4 - 2C_4, & Q_S \phi_S + Q_C \phi_C &\sim 2O_4 - S_4, \\ Q_O \phi_V + Q_V \phi_O &\sim 4O_4 - 2S_4, & Q_S \phi_C + Q_C \phi_S &\sim \text{massive},\end{aligned}$$

from which one can easily read the spectrum of the  $Z_2$  orbifold of the type IIB superstring. The untwisted sector comprises the  $\mathcal{N} = (2, 0)$  supergravity multiplet and five tensor multiplets, while the twisted sector comprises 16 more tensor multiplets, one from each fixed point. Thus, the full spectrum is anomaly-free and corresponds to the compactification on a smooth K3.

It has been shown in [7,11] that, in the presence of a quantised  $B_{ab}$ , not all fixed points have the same  $\Omega$ -eigenvalue. For instance, for the 2d toroidal compactification of type I' in the presence of a non-trivial background for the antisymmetric tensor, three of the four fixed points have a positive eigenvalue, whereas the fourth one has a negative eigenvalue<sup>†</sup>. The generalisation to the case of a  $T^4/Z_2$  orbifold is straightforward and, for a generic  $B_{ab}$  with rank  $r$ , the number of positive and negative eigenvalues is

$$n_{\pm} = 2^3 (1 \pm 2^{-r/2}). \quad (3.3)$$

As we shall see in a moment, this is of crucial importance in order to obtain a consistent transverse Klein bottle amplitude.

In orbifold compactifications, states coming from twisted sectors are localised on (invariant combinations of) fixed points. As a result, in computing the Klein-bottle amplitude

$$\mathcal{K} = \frac{1}{2} \text{tr}_{\mathcal{H}_c} \Omega q^{L_0} \bar{q}^{\bar{L}_0}$$

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<sup>†</sup> The  $\Omega$ -eigenvalues of the fixed points are strictly related to the signs  $\gamma_{\epsilon}$  that we have discussed in the previous section.

one has to combine the action of world-sheet parity on closed-string states with the action of world-sheet parity on fixed points. Thus, taking into account (3.3), from (3.2), one can deduce the following expression for the Klein-bottle amplitude restricted to the low-lying excitations:

$$\begin{aligned} \mathcal{K}_0^{(r)} \sim \frac{1}{2} \Big\{ & (Q_O \phi_O + Q_V \phi_V) + (Q_O \phi_V + Q_V \phi_O) + \\ & + (n_+ - n_-) [(Q_S \phi_S + Q_C \phi_C) + (Q_S \phi_C + Q_C \phi_S)] \Big\}, \end{aligned}$$

from which one can immediately read the spectrum of massless excitations. Beside the  $\mathcal{N} = (1, 0)$  gravitational multiplet coupled to the universal tensor multiplet and four hypermultiplets from the untwisted sector, one gets  $n_+$  hypermultiplets and  $n_-$  tensor multiplets from the twisted sector, as a result of the different behaviour of the  $Z_2$  fixed points under  $\Omega$ , *i.e.* of the different charge of the orientifold planes.

The full Klein-bottle amplitude can be computed adding the contributions from massive states filling a sublattice of the original  $T^4$ . Due to the  $Z_2$  orbifolding, both momentum and winding states contribute to the amplitude. Actually, the latter have to satisfy the constraint (2.2). As a result, one has to introduce a projector in the sum over winding states, so that the full amplitude reads

$$\begin{aligned} \mathcal{K}^{(r)} = & \frac{1}{4} (Q_O + Q_V) (q^2) \left[ \sum_m \frac{q^{\frac{\alpha'}{2} m^T g^{-1} m}}{\eta^4(q^2)} + 2^{-4} \sum_{\epsilon=0,1} \sum_n \frac{q^{\frac{1}{2\alpha'} n^T g n} e^{\frac{2i\pi}{\alpha'} n^T B \epsilon}}{\eta^4(q^2)} \right] \\ & + \frac{2^{(4-r)/2}}{2} (Q_S + Q_C) (q^2) \left( \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4} \right) (q^2). \end{aligned}$$

The coefficient in front of the winding sum ensures the correct normalisation of the graviton. After an  $S$  modular transformation, one gets the following expression for the transverse channel Klein-bottle amplitude:

$$\begin{aligned} \tilde{\mathcal{K}} = & \frac{2^5}{4} (Q_O + Q_V) (i\ell) \left[ \text{Vol} \sum_n \frac{(e^{-2\pi\ell})^{\frac{1}{\alpha'} n^T g n}}{\eta^4(i\ell)} + \right. \\ & \left. + \frac{2^{-4}}{\text{Vol}} \sum_{\epsilon=0,1} \sum_m \frac{(e^{-2\pi\ell})^{\alpha' (m + \frac{1}{\alpha'} B \epsilon)^T g^{-1} (m + \frac{1}{\alpha'} B \epsilon)}}{\eta^4(i\ell)} \right] + \\ & + \frac{2^{5-r/2}}{2} (Q_O - Q_V) (i\ell) \left( \frac{\vartheta_3^2 \vartheta_4^2}{\eta^4} \right) (i\ell), \end{aligned}$$

where

$$\text{Vol} = \sqrt{\det(g/\alpha')}$$

denotes the volume of the four dimensional internal manifold. Extracting the leading contributions to the tadpoles, one can show that  $\tilde{\mathcal{K}}$  effectively involves states with coefficients that are perfect squares as required by the consistency of the two-dimensional conformal field theory in the presence of boundaries and/or crosscaps [2,4]

$$\begin{aligned} \tilde{\mathcal{K}}_0^{(r)} = \frac{2^5}{4} & \left[ (Q_O \phi_O + Q_V \phi_V) \left( \sqrt{\text{Vol}} + \frac{2^{-r/2}}{\sqrt{\text{Vol}}} \right)^2 + \right. \\ & \left. + (Q_O \phi_V + Q_V \phi_O) \left( \sqrt{\text{Vol}} - \frac{2^{-r/2}}{\sqrt{\text{Vol}}} \right)^2 \right]. \end{aligned} \quad (3.4)$$

Before turning to the open sector, let us pause for a moment and comment on the action of the NS-NS antisymmetric tensor on the twisted sector. Although it is evident that a non-vanishing  $B$ -flux modifies the lattice sum, projecting on suitable winding states that satisfy (2.2), less obvious is the fact that it alters the structure of the twisted sector, that does not depend on the moduli defining the size and the shape of the lattice, and *a priori* does not know anything about the  $B_{ab}$  background. Still, simply imposing the “rule of perfect squares” on the crosscap-to-crosscap reflection coefficients for closed-string states one would have discovered that, effectively,  $B_{ab}$  alters the  $\Omega$ -projection on the fixed points. In this way, one can easily obtain the correct parametrisation, even without resorting to a geometrical picture of the orbifold model, an option clearly of interest, for instance, for asymmetric orbifolds. The same procedure can then be applied to the open-string annulus and Möbius amplitudes that, in the transverse channel, have to satisfy similar constraints.

We can now proceed to the construction of the open descendants, introducing the open sector. From the torus amplitude (3.2) and from our knowledge of the structure of the fixed points, we can write the contributions of massless states to the annulus amplitude

$$\begin{aligned} \tilde{\mathcal{A}}_0^{(r)} = \frac{2^{-5}}{4} & \left\{ (Q_O \phi_O + Q_V \phi_V) \left( 2^{r/2} \sqrt{\text{Vol}} I_N + \frac{1}{\sqrt{\text{Vol}}} \sum_{i=1}^{2^{4-r}} I_D^i \right)^2 + \right. \\ & + (Q_O \phi_V + Q_V \phi_O) \left( 2^{r/2} \sqrt{\text{Vol}} I_N - \frac{1}{\sqrt{\text{Vol}}} \sum_{i=1}^{2^{4-r}} I_D^i \right)^2 + \\ & + 16 \sum_{i=1}^{2^{4-r}} \left[ (Q_S \phi_S + Q_C \phi_C) \left( 2^{(r-4)/2} R_N - R_D^i \right)^2 + \right. \\ & \left. \left. + (Q_S \phi_C + Q_C \phi_S) \left( 2^{(r-4)/2} R_N + R_D^i \right)^2 \right] \right\}, \end{aligned} \quad (3.5)$$

where we have already related the boundary-to-boundary reflection coefficients to the Chan-Paton multiplicities. Here  $I$  denotes the sum of Chan-Paton charges, while  $R$  parametrises the orbifold-induced gauge symmetry breaking. The indices  $N$  and  $D$  refer to Neumann (D9-branes) and Dirichlet (D5-branes) charges, respectively. Introducing the contributions from momentum and (projected) winding massive states, one obtains the following expressions for the annulus amplitude in the transverse channel

$$\begin{aligned}\tilde{\mathcal{A}}^{(r)} = & \frac{2^{-5}}{4} \left\{ (Q_O + Q_V)(i\ell) \left[ 2^{r-4} \text{Vol} I_N^2 \sum_{\epsilon=0,1} \sum_n \frac{(e^{-2\pi\ell})^{\frac{1}{4\alpha'}} n^T g n e^{\frac{2i\pi}{\alpha'} n^T B \epsilon}}{\eta^4(i\ell)} + \right. \right. \\ & \left. \left. + \frac{1}{\text{Vol}} \sum_{i,j=1}^{2^{4-r}} I_D^i I_D^j \sum_m \frac{(e^{-2\pi\ell})^{\frac{\alpha'}{4}} m^T g^{-1} m e^{2i\pi m^T (x^i - x^j)}}{\eta^4(i\ell)} \right] + \right. \\ & + 2 \times 2^{r/2} (Q_O - Q_V)(i\ell) \left( \frac{\vartheta_3^2 \vartheta_4^2}{\eta^4} \right) (i\ell) \sum_{i=1}^{2^{4-r}} I_N I_D^i + \\ & + 4(Q_S + Q_C)(i\ell) \left( \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4} \right) (i\ell) \left[ R_N^2 + \sum_{i=1}^{2^{4-r}} (R_D^i)^2 \right] + \\ & \left. - 2 \times 2^{r/2} (Q_S - Q_C)(i\ell) \left( \frac{\vartheta_2^2 \vartheta_4^2}{\eta^4} \right) (i\ell) \sum_{i=1}^{2^{4-r}} R_N R_D^i \right\}\end{aligned}$$

and in the direct channel

$$\begin{aligned}\mathcal{A}^{(r)} = & \frac{1}{4} \left\{ (Q_O + Q_V)(\sqrt{q}) \left[ 2^{r-4} I_N^2 \sum_{\epsilon=0,1} \sum_m \frac{q^{\frac{\alpha'}{2} (m + \frac{1}{\alpha'} B \epsilon)^T g^{-1} (m + \frac{1}{\alpha'} B \epsilon)}}{\eta^4(\sqrt{q})} + \right. \right. \\ & \left. \left. + \sum_{i,j=1}^{2^{4-r}} I_D^i I_D^j \sum_n \frac{q^{\frac{1}{2\alpha'} (n + \frac{1}{2} (x^i - x^j))^T g (n + \frac{1}{2} (x^i - x^j))}}{\eta^4(\sqrt{q})} \right] + \right. \\ & + (Q_O - Q_V)(\sqrt{q}) \left( \frac{\vartheta_3^2 \vartheta_4^2}{\eta^4} \right) (\sqrt{q}) \left[ R_N^2 + \sum_{i=1}^{2^{4-r}} (R_D^i)^2 \right] + \\ & + \frac{2^{r/2}}{2} (Q_S + Q_C)(\sqrt{q}) \left( \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4} \right) (\sqrt{q}) \sum_{i=1}^{2^{4-r}} I_N I_D^i + \\ & \left. + \frac{2^{r/2}}{2} (Q_S - Q_C)(\sqrt{q}) \left( \frac{\vartheta_2^2 \vartheta_4^2}{\eta^4} \right) (\sqrt{q}) \sum_{i=1}^{2^{4-r}} R_N R_D^i \right\}.\end{aligned}\tag{3.6}$$

From this last expression, one can anticipate that states in the twisted sector will have multiplicities related to the rank of the NS-NS antisymmetric tensor. Once more, this

non-trivial feature emerges naturally, imposing that the boundary-to-boundary reflection coefficients be perfect squares.

To conclude the construction of the open descendants, one has to add the contribution of the Möbius amplitude. From (3.4) and (3.5), we can deduce the terms at the origin of the lattices

$$\begin{aligned}\tilde{\mathcal{M}}_0^{(r)} = & -\frac{2}{4} \left[ (\hat{Q}_O \hat{\phi}_O + \hat{Q}_V \hat{\phi}_V) \left( \sqrt{\text{Vol}} + \frac{2^{-r/2}}{\sqrt{\text{Vol}}} \right) \left( 2^{r/2} \sqrt{\text{Vol}} I_N + \frac{1}{\sqrt{\text{Vol}}} \sum_{i=1}^{2^{4-r}} I_D^i \right) + \right. \\ & \left. + (\hat{Q}_O \hat{\phi}_V + \hat{Q}_V \hat{\phi}_O) \left( \sqrt{\text{Vol}} - \frac{2^{-r/2}}{\sqrt{\text{Vol}}} \right) \left( 2^{r/2} \sqrt{\text{Vol}} I_N - \frac{1}{\sqrt{\text{Vol}}} \sum_{i=1}^{2^{4-r}} I_D^i \right) \right]\end{aligned}$$

that, together with momentum and winding massive states, arrange in

$$\begin{aligned}\tilde{\mathcal{M}}^{(r)} = & -\frac{2}{4} \left\{ (\hat{Q}_O + \hat{Q}_V) (i\ell + \frac{1}{2}) \left[ 2^{(r-4)/2} \text{Vol} I_N \sum_{\epsilon=0,1} \sum_n \frac{(e^{-2\pi\ell})^{\frac{1}{\alpha'}} n^T g n e^{\frac{2i\pi}{\alpha'} n^T B \epsilon} \gamma_\epsilon}{\hat{\eta}^4(i\ell + \frac{1}{2})} + \right. \right. \\ & \left. + \frac{2^{-2}}{\text{Vol}} \sum_{i=1}^{2^{4-r}} I_D^i \sum_{\epsilon=0,1} \sum_m \frac{(e^{-2\pi\ell})^{\alpha' (m + \frac{1}{\alpha'} B \epsilon)^T} g^{-1} (m + \frac{1}{\alpha'} B \epsilon) \tilde{\gamma}_\epsilon}{\hat{\eta}^4(i\ell + \frac{1}{2})} \right] + \\ & \left. + (\hat{Q}_O - \hat{Q}_V) (i\ell + \frac{1}{2}) \left( \frac{\hat{\vartheta}_3^2 \hat{\vartheta}_4^2}{\hat{\eta}^4} \right) (i\ell + \frac{1}{2}) \left( I_N + \sum_{i=1}^{2^{4-r}} I_D^i \right) \right\}.\end{aligned}$$

Notice that both the momentum and the winding sums in the Möbius amplitude depend on  $B_{ab}$ . This is expected, since in the transverse channel  $\tilde{\mathcal{K}}$ ,  $\tilde{\mathcal{A}}$  and  $\tilde{\mathcal{M}}$  must factorise. There is another very important point to stress. The Möbius amplitude involves signs  $\gamma_\epsilon$  and  $\tilde{\gamma}_\epsilon$  that give a correct normalisation to the projectors onto states that satisfy (2.2), similarly to what happens in toroidal compactifications. A  $P$  modular transformation then gives the Möbius amplitude in the direct channel

$$\begin{aligned}\mathcal{M}^{(r)} = & -\frac{1}{4} \left\{ (\hat{Q}_O + \hat{Q}_V) (-\sqrt{q}) \left[ 2^{(r-4)/2} I_N \sum_{\epsilon=0,1} \sum_m \frac{q^{\frac{\alpha'}{2} (m + \frac{1}{\alpha'} B \epsilon)^T} g^{-1} (m + \frac{1}{\alpha'} B \epsilon) \gamma_\epsilon}{\hat{\eta}^4(-\sqrt{q})} + \right. \right. \\ & \left. + 2^{-2} \sum_{i=1}^{2^{4-r}} I_D^i \sum_{\epsilon=0,1} \sum_n \frac{q^{\frac{1}{2\alpha'} n^T g n e^{\frac{2i\pi}{\alpha'} n^T B \epsilon}} \tilde{\gamma}_\epsilon}{\hat{\eta}^4(-\sqrt{q})} \right] + \\ & \left. - (\hat{Q}_O - \hat{Q}_V) (-\sqrt{q}) \left( \frac{\hat{\vartheta}_3^2 \hat{\vartheta}_4^2}{\hat{\eta}^4} \right) (-\sqrt{q}) \left( I_N + \sum_{i=1}^{2^{4-r}} I_D^i \right) \right\}\end{aligned}\tag{3.7}$$

that completes the construction of the open descendants.

#### 4. Tadpoles, discrete Wilson lines and anomalies

We are now ready to extract the massless contributions to the transverse-channel amplitudes, and thus to read the tadpoles for the unphysical states. The tadpole conditions are equivalent to those of the standard  $Z_2$  orbifold without  $B_{ab}$  [3,10], aside from the fact that now the total size of the gauge group is reduced according to the rank of the NS-NS antisymmetric tensor. One thus gets

$$\sqrt{\text{Vol}} \left( 2^5 - 2^{r/2} I_N \right) \pm \frac{1}{\sqrt{\text{Vol}}} \left( 2^{5-r/2} - \sum_{i=1}^{n_+ - n_-} I_D^i \right) = 0$$

from untwisted states, and

$$2^{(r-4)/2} R_N - R_D^i = 0, \quad \text{for } i = 1, \dots, 2^{4-r},$$

from twisted states.

We have now all the ingredients needed to extract the spectrum of massless excitations. We have already described the closed spectrum in the previous section. It comprises the  $\mathcal{N} = (1, 0)$  supergravity multiplet coupled to  $1 + n_-$  tensor multiplets and  $4 + n_+$  hypermultiplets. In order to extract the massless open spectrum, we expand the amplitudes (3.6) and (3.7) to lowest order in  $q$ , obtaining

$$\begin{aligned} \mathcal{A}_0^{(r)} \sim & \frac{1}{4} \left( I_N^2 + R_N^2 + \sum_{i=1}^{2^{4-r}} (I_D^i)^2 + (R_D^i)^2 \right) (Q_O \phi_O + Q_V \phi_V) + \\ & + \frac{1}{4} \left( I_N^2 - R_N^2 + \sum_{i=1}^{2^{4-r}} (I_D^i)^2 - (R_D^i)^2 \right) (Q_O \phi_V + Q_V \phi_O) + \\ & + \frac{2^{r/2}}{2} \sum_{i=1}^{2^{4-r}} (I_N I_D^i + R_N R_D^i) (Q_S \phi_S + Q_C \phi_C) + \\ & + \frac{2^{r/2}}{2} \sum_{i=1}^{2^{4-r}} (I_N I_D^i - R_N R_D^i) (Q_S \phi_C + Q_C \phi_S) \end{aligned}$$

for the annulus amplitude, and

$$\begin{aligned} \mathcal{M}_0^{(r)} \sim & -\frac{1}{4} (\hat{Q}_O + \hat{Q}_V) (\hat{\phi}_O + \hat{\phi}_V) \left( \gamma_0 I_N + \tilde{\gamma}_0 \sum_{i=1}^{2^{4-r}} I_D^i \right) + \\ & + \frac{1}{4} (\hat{Q}_O - \hat{Q}_V) (\hat{\phi}_O - \hat{\phi}_V) \left( I_N + \sum_{i=1}^{2^{4-r}} I_D^i \right) \end{aligned}$$

for the Möbius amplitude. In order to read the spectrum from the above expressions, we must still introduce an explicit parametrisation of  $I_\alpha$  and  $R_\alpha$  in terms of Chan-Paton multiplicities. It is naturally obtained demanding a consistent particle interpretation of the physical spectrum, *i.e.* imposing that the Möbius amplitude symmetrises correctly the annulus amplitude. It is then evident that the signs  $\gamma_0$  and  $\tilde{\gamma}_0$  present in  $\mathcal{M}$  play a crucial role. In order to respect the reality of the annulus amplitude and the structure of the transverse channel, they have to coincide, *i.e.*  $\gamma_0 = \tilde{\gamma}_0$ , and then we are left with two possibilities:  $\gamma_0 = \tilde{\gamma}_0 = +1$  and  $\gamma_0 = \tilde{\gamma}_0 = -1$ . In the former case, the Möbius amplitude contains at the massless level only the untwisted hypermultiplet. As a result, the gauge group is unitary and one is led to the following parametrisation in terms of complex Chan-Paton charges:

$$\begin{aligned} I_N &= N + \bar{N}, & R_N &= i(N - \bar{N}), \\ I_D^i &= D^i + \bar{D}^i, & R_D^i &= i(D^i - \bar{D}^i). \end{aligned}$$

This is consistent with the well known result for the  $T^4/Z_2$  orbifold [3,10] with vanishing  $B$ -field, for which the signs  $\gamma_0$  and  $\tilde{\gamma}_0$  are uniquely fixed by the tadpole conditions. The massless spectrum then comprises non-abelian vector multiplets with gauge group

$$G_{\text{CP}} = \text{U}(2^{4-r/2})\Big|_{99} \otimes \text{U}(2^{4-r/2})\Big|_{55}$$

and additional charged hypermultiplets in the representations

$$2(A; 1) \oplus 2(1; A) \oplus 2^{r/2}(F; F),$$

where  $F$  ( $A$ ) denotes the fundamental (antisymmetric) representation. Here and in the following we have decided to confine all the D5-branes to the same fixed point. In general, however, it is possible to distribute them at different fixed points or at any generic positions in the internal manifold, thus breaking the gauge group to a product of unitary and/or symplectic subgroups. In table 1 we summarise the massless spectra for the various choices of  $r = 0, 2, 4$ .

$r$	$n_T^c$	$n_H^c$	$G_{\text{CP}}$	open hypermultiplets
0	1	20	$\text{U}(16)_{99} \otimes \text{U}(16)_{55}$	$2(120; 1) \oplus 2(1; 120) \oplus (16; 16)$
2	5	16	$\text{U}(8)_{99} \otimes \text{U}(8)_{55}$	$2(28; 1) \oplus 2(1; 28) \oplus 2(8; 8)$
4	7	14	$\text{U}(4)_{99} \otimes \text{U}(4)_{55}$	$2(6; 1) \oplus 2(1; 6) \oplus 4(4; 4)$

**Table 1.** Massless spectrum for  $\gamma_0 = \tilde{\gamma}_0 = +1$ .

When  $r \neq 0$ , one has the additional choice  $\gamma_0 = \tilde{\gamma}_0 = -1$ . In this case, the Möbius amplitude contains the massless vector with a positive sign, and therefore the contribution of the annulus amplitude is symmetrised<sup>§</sup>. The gauge groups are thus symplectic, and call for the following parametrisation in terms of real Chan-Paton charges:

$$\begin{aligned} I_N &= N_1 + N_2, & R_N &= N_1 - N_2, \\ I_D^i &= D_1^i + D_2^i, & R_D^i &= D_1^i - D_2^i. \end{aligned}$$

As a result, one finds a generic gauge group of the form

$$G_{\text{CP}} = \text{Sp}(2^{4-r/2}) \otimes \text{Sp}(2^{4-r/2}) \Big|_{99} \otimes \text{Sp}(2^{4-r/2}) \otimes \text{Sp}(2^{4-r/2}) \Big|_{55}$$

with charged hypermultiplets in bi-fundamentals. In table 2 we summarise the resulting massless spectra for the choices  $r = 2, 4$ .

$r$	$n_T^c$	$n_H^c$	$G_{\text{CP}}$	open hypermultiplets
2	5	16	$\text{Sp}(8)_{99}^2 \otimes \text{Sp}(8)_{55}^2$	$(8, 8; 1, 1) \oplus (1, 1; 8, 8)$ $\oplus (8, 1; 8, 1) \oplus (1, 8; 1, 8)$
4	7	14	$\text{Sp}(4)_{99}^2 \otimes \text{Sp}(4)_{55}^2$	$(4, 4; 1, 1) \oplus (1, 1; 4, 4)$ $\oplus 2(4, 1; 4, 1) \oplus 2(1, 4; 1, 4)$

**Table 2.** Massless spectrum for  $\gamma_0 = \tilde{\gamma}_0 = -1$ .

It is not surprising to realize that all these models had been already discovered in [2,19], using the rational algorithm. In fact, group lattices naturally involve a non-vanishing quantised  $B$ -field given explicitly by the adjacency matrix of the associated algebra. In particular, the  $Z_2$  models discussed in [2,19] were built starting from a toroidal compactification on the  $\text{SO}(4)^2$  lattice and on the  $\text{SO}(8)$  lattice. The latter has a  $B_{ab}$  with  $r = 2$  and, indeed, the spectrum of the open descendants associated to the (geometrical) diagonal modular invariant partition function is exactly the one reported in table 2. It is quite surprising, however, to see how the second model is obtained in the rational construction. Actually, it is related to a different modular invariant combination of characters for the same  $\text{SO}(8)$  lattice that, as we have seen, involves naively a rank-two antisymmetric tensor.

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<sup>§</sup> See also [18] for a previous discussion about the appearance of symplectic gauge groups in the  $T^4/Z_2$  orbifold.



Actually, this is true for the geometrical combination of characters corresponding to the diagonal (or, better, charge-conjugation) modular invariant

$$|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2,$$

while all other non-geometrical choices, as for instance

$$|O_8|^2 + |V_8|^2 + S_8 \bar{C}_8 + C_8 \bar{S}_8,$$

effectively increase the rank of the  $B_{ab}$  matrix.

Actually, it was shown in [2] that the models in table 1 can also be obtained in the rational construction. They involve the introduction of discrete Wilson lines in the Möbius amplitude, that have the effect to modify the  $P$  transformation. In order to illustrate this phenomenon, let us consider the models with  $r = 2$ . In the rational model, the transverse-channel Möbius amplitude is [2]

$$\tilde{\mathcal{M}} = -\alpha(\hat{Q}_O \hat{O}_4 \hat{O}_4 - \hat{Q}_V \hat{V}_4 \hat{V}_4).$$

Since  $P$  acts as  $\sigma_1$  on  $(\hat{O}_4, \hat{V}_4)$ , with  $\sigma_1$  the first Pauli matrix, it is evident that the direct-channel amplitude

$$\mathcal{M} = \alpha(\hat{Q}_O \hat{O}_4 \hat{O}_4 - \hat{Q}_V \hat{V}_4 \hat{V}_4)$$

calls for symplectic gauge groups. Actually, this is not the only possible choice. Compatibly with the supercurrent, one may introduce discrete Wilson lines, changing the definition of the internal characters

$$\begin{aligned} \hat{O}_4 &= \hat{O}_2 \hat{O}_2 - \hat{V}_2 \hat{V}_2 \rightarrow \hat{O}'_4 = \hat{O}_2 \hat{O}_2 + \hat{V}_2 \hat{V}_2, \\ \hat{V}_4 &= \hat{O}_2 \hat{V}_2 + \hat{V}_2 \hat{O}_2 \rightarrow \hat{V}'_4 = \hat{O}_2 \hat{V}_2 - \hat{V}_2 \hat{O}_2, \end{aligned}$$

and thus affecting the  $P$  transformation, whose action on the primed characters  $(\hat{O}'_4, \hat{V}'_4)$  is now represented by the Pauli matrix  $\sigma_3$ . As a result, the new

$$\tilde{\mathcal{M}}' = -\alpha(\hat{Q}_O \hat{O}'_4 \hat{O}'_4 - \hat{Q}_V \hat{V}'_4 \hat{V}'_4)$$

maps to

$$\mathcal{M}' = -\alpha(\hat{Q}_V \hat{O}'_4 \hat{O}'_4 - \hat{Q}_O \hat{V}'_4 \hat{V}'_4)$$

and calls for unitary gauge groups. We are thus led to identify the action of discrete Wilson lines for non-zero  $B_{ab}$  with the signs  $\gamma_\epsilon$  and  $\tilde{\gamma}_\epsilon$  present in the irrational construction.

As a side remark, let us recall how the  $U(16) \otimes U(16)$  model was originally built in [2]. It originates from the compactification on the  $SO(4)^2$  lattice, once the residual internal global  $SO(2)^4$  symmetry is broken, resorting to eight copies of the Ising model. As a result, all characters become real and contribute to the transverse annulus amplitude, thus enhancing the overall size of the Chan-Paton gauge group. At this point, as in the previous case, the introduction of discrete Wilson lines turns each pair of Sp gauge groups into a single unitary group, thus yielding the  $U(16) \otimes U(16)$  model.

To close this section, let us comment on the cancellation of the residual anomalies for these models. As usual, tadpole cancellations guarantee the vanishing of the irreducible  $\text{tr} R^4$  and  $\text{tr} F_\alpha^4$  terms. However, this is not in general sufficient, and some other mechanisms are required in order to compensate the reducible terms. In  $D = 10$  this is the familiar Green-Schwarz mechanism, that involves the universal 2-form present in the spectrum [20]. This mechanism still applies to any compactification with a single 2-form, for instance for the K3 reduction of the heterotic string, or for the  $T^4/Z_2$  reduction of the type-I string with a vanishing background for the NS-NS  $B_{ab}$ . However, when additional tensor fields are present (a generic feature of six-dimensional open-string vacua), the cancellation of gauge and gravitational anomalies requires a generalised Green-Schwarz mechanism [21]. In fact, for  $n_T$  tensor multiplets the residual anomaly polynomial takes the form<sup>‡</sup>

$$\mathcal{I}_8 = - \sum_{x,y} c_x^r c_y^s \eta_{rs} \text{tr}_x F^2 \text{tr}_y F^2 ,$$

with the  $c$ 's a collection of constants and  $\eta$  the Minkowski metric for  $SO(1, n_T)$ . Accordingly, the field strengths of the 2-forms include suitable combinations of (Yang-Mills and gravitational) Chern-Simons forms, and the kinetic term for the gauge vectors involves couplings to the scalars  $v_r$  in the tensor multiplets:

$$e^{-1} \mathcal{L} \sim -\frac{1}{2} v_r c^{rz} \text{tr}_z F_{\mu\nu} F^{\mu\nu} ,$$

with corresponding generic strong-coupling singularities [21] that may be related to the appearance of tensionless strings [22]. Actually, it is not true that the presence of additional tensor fields in the spectrum automatically calls for a generalised Green-Schwarz

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<sup>‡</sup> In this expression, for  $r = s = 0$  the indices  $x$  and  $y$  refer to the contribution of the gravitational term  $\text{tr} R^2$ . In these supersymmetric models, the self-dual 2-form present in the gravitational multiplet remove the residual gravitational and mixed anomalies.

mechanism involving all of them. For instance, let us consider the models reported in table 1. In this case the residual anomaly polynomial is

$$\begin{aligned} \mathcal{I}_8^{(r)} = & -\frac{2^{2/r}}{16} \left( 2^{-r/2} \text{tr} R^2 - \text{tr} F_1^2 - \text{tr} F_2^2 \right)^2 + \frac{2^{r/2}}{16} (\text{tr} F_1^2 - \text{tr} F_2^2)^2 + \\ & + \frac{2^{r/2}}{6} \text{tr} F_1 \left( \frac{1}{16} \text{tr} R^2 \text{tr} F_2 - \text{tr} F_2^3 \right) + \frac{2^{r/2}}{6} \text{tr} F_2 \left( \frac{1}{16} \text{tr} R^2 \text{tr} F_1 - \text{tr} F_1^3 \right). \end{aligned} \quad (4.1)$$

From this expression, one can read that, besides the self-dual 2-form present in the supergravity multiplet, only the untwisted antiself-dual 2-form is involved in the Green-Schwarz mechanism (aside from twisted scalar fields responsible for the cancellation of abelian anomalies). This has been explicitly checked in [23,24,25] for the  $r = 0$  case, where this result is expected since only the untwisted tensor multiplet is available. Nonetheless, one can use these results to justify the structure of (4.1) for generic  $r$ . In fact, for the case  $\gamma_0 = \tilde{\gamma}_0 = +1$ , the explicit value of the constants  $c$  is<sup>\*</sup> [23]

$$c \sim (N - \bar{N}),$$

as can be seen turning on a background magnetic field. Thus, due to the numerical identification of conjugate charges, there is effectively no coupling between twisted scalars and vectors and, as a result, only a pair of tensors is involved in the generalised Green-Schwarz mechanism. The situation changes drastically for the choice  $\gamma_0 = \tilde{\gamma}_0 = -1$ . In this case, each  $U(N)$  group splits in two different symplectic groups  $\text{Sp}(N_1) \otimes \text{Sp}(N_2)$ , and the constants  $c$  associated to the  $\alpha$ -th factor in the Chan-Paton gauge group are now given by

$$c \sim N_\alpha.$$

Thus, they do not vanish, as expected from the corresponding expressions for the anomaly polynomials:

$$\begin{aligned} \mathcal{I}_8'^{(r)} = & -\frac{2^{r/2}}{64} \left( 2^{(2-r)/2} \text{tr} R^2 - \text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_3^2 - \text{tr} F_4^2 \right)^2 + \\ & + \frac{2^{r/2}}{64} (\text{tr} F_1^2 + \text{tr} F_2^2 - \text{tr} F_3^2 - \text{tr} F_4^2)^2 + \\ & + \frac{2^{r/2}}{64} (2 - 2^{(r-2)/2}) (\text{tr} F_1^2 - \text{tr} F_2^2 + \text{tr} F_3^2 - \text{tr} F_4^2)^2 + \\ & + \frac{2^{r/2}}{64} (4 - 2^{(r-2)/2}) (\text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_3^2 + \text{tr} F_4^2)^2 \end{aligned}$$

that clearly involve twisted antiself-dual 2-forms.

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<sup>\*</sup> Here we are only interested in the dependence of the constants on the Chan-Paton charges. For a proper normalisation, we refer to [23].

## 5. The $T^6/Z_3$ orbifold with a non-vanishing $B_{ab}$

In this section we want to apply our previous results to the four-dimensional  $Z$  orbifold, thus generalising [12] (see also [26]). Since the orbifold group does not contain any generator that squares to the identity, it is natural to expect that this construction does not present any new interesting features. As we shall see, however, this is not the case, since some subtleties are indeed present in the construction of the Möbius amplitude. In what follows we will use the notation introduced in [12].

The building blocks for the  $Z$  orbifold are the amplitudes ( $\omega = e^{2i\pi/3}$ ,  $\rho, \lambda = 0, \pm 1$ )

$$\Xi_{\rho,\lambda}(q) = \left( \frac{A_0 \chi_\rho + \omega^\lambda A_+ \chi_{\rho-1} + \bar{\omega}^\lambda A_- \chi_{\rho+1}}{H_{\rho,\lambda}^3} \right) (q),$$

where

$$H_{0,\lambda}(q) = q^{\frac{1}{12}} \prod_{n=1}^{\infty} (1 - \omega^\lambda q^n)(1 - \bar{\omega}^\lambda q^n),$$

$$H_{+,\lambda}(q) = H_{-,-\lambda}(q) = \frac{1}{\sqrt{3}} q^{-\frac{1}{36}} \prod_{n=0}^{\infty} (1 - \omega^\lambda q^{n+\frac{1}{3}})(1 - \bar{\omega}^\lambda q^{n+\frac{2}{3}})$$

denote the contribution of the internal bosons, while the level one  $SU(3)$  characters  $\chi_{0,\pm}$  and the combinations\*

$$A_\rho = V_2 \xi_{4\rho} + O_2 \xi_{4\rho+6} - S_2 \xi_{4\rho-3} - C_2 \xi_{4\rho+3}$$

denote the contribution of the world-sheet fermions. Thus, the torus amplitude for the type IIB string is

$$\mathcal{T} = \frac{1}{3} \left[ \Xi_{0,0} \bar{\Xi}_{0,0} \sum_{m,n} q^{\frac{\alpha'}{4} p^T g^{-1} p} \bar{q}^{\frac{\alpha'}{4} \tilde{p}^T g^{-1} \tilde{p}} + \sum_{\lambda=\pm 1} \Xi_{0,\lambda} \bar{\Xi}_{0,-\lambda} + \sum_{\rho=\pm 1} \sum_{\lambda=0,\pm 1} \Xi_{\rho,\lambda} \bar{\Xi}_{-\rho,-\lambda} \right].$$

The left and right momenta are given explicitly by (2.1), with the metric  $g_{ab}$  pertaining to three orthogonal copies of a two-dimensional hexagonal lattice, as required by the  $Z_3$  symmetry.

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\* Here  $(O_2, V_2, S_2, C_2)$  are level one  $SO(2)$  characters while  $\xi_m$  ( $m = -5, \dots, 6$ ) are the 12 characters of the  $\mathcal{N} = 2$  superconformal model with  $c = 1$ , equivalent to the rational torus at radius  $R = \sqrt{12}$ .

As in toroidal models, the Klein bottle amplitude is not modified by the presence of the NS-NS antisymmetric tensor. It is given by [12]

$$\mathcal{K} = \frac{1}{6} \left[ \Xi_{0,0}(q^2) \sum_m q^{\frac{\alpha'}{2} m^T g^{-1} m} + \sum_{\lambda=\pm 1} \Xi_{0,\lambda}(q^2) \right]$$

and, together with the torus amplitude, leaves at the massless level the  $\mathcal{N} = 1$  supergravity multiplet, a universal chiral multiplet and 9 additional chiral multiplets from the untwisted sector, as well as 27 chiral multiplets from the twisted sectors.

The open-string sector now involves in the transverse-channel the projector on the winding states satisfying (2.2), and results in the following one-loop amplitudes:

$$\mathcal{A} = \frac{1}{6} \left[ N_0^2 2^{r-6} \Xi_{0,0}(\sqrt{q}) \sum_{\epsilon=0,1} \sum_m q^{\frac{\alpha'}{2} (m + \frac{1}{\alpha'} B \epsilon)^T g^{-1} (m + \frac{1}{\alpha'} B \epsilon)} + \sum_{\lambda=\pm 1} N_\lambda^2 \Xi_{0,\lambda}(\sqrt{q}) \right]$$

and

$$\begin{aligned} \mathcal{M} = -\frac{1}{6} \left[ \delta_0 N_0 2^{(r-6)/2} \hat{\Xi}_{0,0}(-\sqrt{q}) \sum_{\epsilon=0,1} \sum_m q^{\frac{\alpha'}{2} (m + \frac{1}{\alpha'} B \epsilon)^T g^{-1} (m + \frac{1}{\alpha'} B \epsilon)} \gamma_\epsilon + \right. \\ \left. + \sum_{\lambda=\pm 1} \delta_\lambda N_\lambda \hat{\Xi}_{0,\lambda} \right], \end{aligned}$$

where

$$N_\lambda = n + \omega^\lambda m + \bar{\omega}^\lambda \bar{m}$$

are suitable combinations of the Chan-Paton multiplicities  $n, m, \bar{m}$ . The Möbius amplitude involves different kinds of signs  $\delta_\lambda$  and  $\gamma_\epsilon$ . The latter, by now familiar, enforce a proper normalisation of the Möbius amplitude, whereas the former are fixed by the tadpole conditions

$$\begin{aligned} n + m + \bar{m} &= 2^{5-r/2} \delta_0 \\ n + \omega^\lambda m + \bar{\omega}^\lambda \bar{m} &= -4 \delta_\lambda \quad (\lambda = \pm 1) \end{aligned}$$

whose solution requires  $\delta_0 = +1$  and  $\delta_\pm = +1$  ( $\delta_\pm = -1$ ) for  $r = 0, 4$  ( $r = 2, 6$ ). In order to better appreciate the role of the signs  $\delta_\pm$  and  $\gamma_\epsilon$ , let us expand the open-string amplitudes keeping only the massless contributions. One has

$$\mathcal{A}_0 \sim \left( \frac{n^2}{2} + m\bar{m} \right) A_0 \chi_0 + \left( \frac{\bar{m}^2}{2} + nm \right) A_+ \chi_- + \left( \frac{m^2}{2} + n\bar{m} \right) A_- \chi_+$$

for the annulus amplitude, and

$$\begin{aligned} \mathcal{M}_0 \sim -\frac{1}{6} \Bigg\{ & \left[ n(\gamma_0 + \delta_+ + \delta_-) + m(\gamma_0 + \omega\delta_+ + \bar{\omega}\delta_-) + \bar{m}(\gamma_0 + \bar{\omega}\delta_+ + \omega\delta_-) \right] \hat{A}_0 \hat{\chi}_0 + \\ & + \left[ n(\gamma_0 + \bar{\omega}\delta_+ + \omega\delta_-) + m(\gamma_0 + \delta_+ + \delta_-) + \bar{m}(\gamma_0 + \omega\delta_+ + \bar{\omega}\delta_-) \right] \hat{A}_+ \hat{\chi}_- + \\ & + \left[ n(\gamma_0 + \omega\delta_+ + \bar{\omega}\delta_-) + m(\gamma_0 + \bar{\omega}\delta_+ + \omega\delta_-) + \bar{m}(\gamma_0 + \delta_+ + \delta_-) \right] \hat{A}_- \hat{\chi}_+ \Bigg\} \end{aligned}$$

for the Möbius amplitude. It is then evident that, in order to have a consistent particle interpretation, the signs  $\gamma_0$  and  $\delta_{\pm}$  are to be the same. Therefore, for the  $Z$  orbifold one does not have the option to introduce discrete Wilson lines for a given  $B_{ab}$ . They are uniquely fixed by the rank of the NS-NS antisymmetric tensor. Some of the gauge groups of reduced rank in [12] should thus be corrected, according to the following table:

$r$	$G_{\text{CP}}$	open chiral multiplets
0	$\text{U}(12) \otimes \text{SO}(8)$	$3 \times (\overline{12}, 8) \oplus 3 \times (66, 1)$
2	$\text{U}(4) \otimes \text{Sp}(8)$	$3 \times (\overline{4}, 8) \oplus 3 \times (10, 1)$
4	$\text{U}(4)$	$3 \times 6$
6	$\text{Sp}(4)$	—

**Table 3.** Massless spectrum for the  $T^6/Z_3$  orbifold.

## 6. Comments on brane supersymmetry breaking

To conclude, let us briefly comment on the results of [13]. The structure of the amplitudes presented in the previous sections is uniquely fixed by the constraints of the underlying two-dimensional conformal field theory and by space-time supersymmetry. This last requirement, for instance, prevents the introduction of signs in the Klein bottle amplitude that are actually allowed by the crosscap constraint [4]. Actually, only one (non-trivial) choice is compatible with supersymmetry

$$\mathcal{K} = \frac{1}{4}(Q_O + Q_V) \left[ \sum_m \frac{(-)^m q^{\frac{\alpha'}{2} m^T g^{-1} m}}{\eta^4} + \sum_n \frac{(-)^n q^{\frac{1}{2\alpha'} n^T g n}}{\eta^4} \right] + \frac{1}{8}(8-8)(Q_S + Q_C) \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4}$$

that results in a model with 9 tensor multiplets, 12 hypermultiplets and *no* open strings.

The nice observation of [13] consists in relaxing the requirement of space-time supersymmetry, thus allowing for other consistent solutions of the crosscap constraint. In particular, they considered the case in which the whole twisted sector is antisymmetrised in the NS-NS sector and symmetrised in the R-R one, yielding an  $\mathcal{N} = (1, 0)$  supersymmetric closed (bulk) spectrum with 17 tensor multiplets and 4 hypermultiplets. The corresponding open spectrum is non-supersymmetric, with non-abelian vectors in the adjoint of  $\text{SO}(16) \otimes \text{SO}(16) \otimes \text{Sp}(16) \otimes \text{Sp}(16)$  and additional fermions and scalars in (anti)symmetric and bi-fundamental representations. As a result of supersymmetry breaking, the Möbius amplitude generates a one-loop cosmological constant supported on the  $\text{D}\bar{5}$ -branes and, consequently, a potential for the NS-NS moduli appears. This is precisely the opposite to what found in previous type I settings, where supersymmetry is spontaneously broken in the bulk but can still be present on the massless excitations [27], or even on the whole massive spectrum [28], of suitable branes. In the following we shall study the modifications induced in this model by a non-vanishing antisymmetric tensor  $B_{ab}$  present in the compactification lattice. For the sake of brevity, we shall only present the various amplitudes, referring to [13] and to the previous sections for a detailed discussion of the subtleties of the construction.

Aside from the torus amplitude (3.1), one has the following contributions:

$$\begin{aligned} \mathcal{K}^{(r)} = & \frac{1}{4}(Q_O + Q_V)(q^2) \left[ \sum_m \frac{q^{\frac{\alpha'}{2} m^T g^{-1} m}}{\eta^4(q^2)} + 2^{-4} \sum_{\epsilon=0,1} \sum_n \frac{q^{\frac{1}{2\alpha'} n^T g n} e^{\frac{2i\pi}{\alpha'} n^T B \epsilon}}{\eta^4(q^2)} \right] + \\ & - \frac{2^{(4-r)/2}}{2} (Q_S + Q_C)(q^2) \left( \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4} \right) (q^2) \end{aligned}$$

from the Klein bottle amplitude,

$$\begin{aligned} \mathcal{A}^{(r)} = & \frac{1}{4} \left\{ (Q_O + Q_V)(\sqrt{q}) \left[ 2^{r-4} I_N^2 \sum_{\epsilon=0,1} \sum_m \frac{q^{\frac{\alpha'}{2} (m + \frac{1}{\alpha'} B \epsilon)^T g^{-1} (m + \frac{1}{\alpha'} B \epsilon)}}{\eta^4(\sqrt{q})} + \right. \right. \\ & \left. \left. + \sum_{i,j}^{2^{4-r}} I_D^i I_D^j \sum_n \frac{q^{\frac{1}{2\alpha'} (n + \frac{1}{2} (x^i - x^j))^T g (n + \frac{1}{2} (x^i - x^j))}}{\eta^4(\sqrt{q})} \right] + \right. \\ & + (Q_O - Q_V)(\sqrt{q}) \left( \frac{\vartheta_3^2 \vartheta_4^2}{\eta^4} \right) (\sqrt{q}) \left( R_N^2 + \sum_{i=1}^{2^{4-r}} (R_D^i)^2 \right) + \\ & + \frac{2^{r/2}}{2} (O_4 S_4 + V_4 C_4 - C_4 O_4 - S_4 V_4)(\sqrt{q}) \left( \frac{\vartheta_2^2 \vartheta_3^2}{\eta^4} \right) (\sqrt{q}) \sum_{i=1}^{2^{4-r}} I_N I_D^i + \\ & \left. - \frac{2^{r/2}}{2} (O_4 S_4 - V_4 C_4 + C_4 O_4 - S_4 V_4)(\sqrt{q}) \left( \frac{\vartheta_2^2 \vartheta_4^2}{\eta^4} \right) (\sqrt{q}) \sum_{i=1}^{2^{4-r}} R_N R_D^i \right\} \end{aligned}$$

from the annulus amplitude, and

$$\begin{aligned}
\mathcal{M}^{(r)} = & -\frac{1}{4} \left\{ 2^{(r-4)/2} I_N (\hat{V}_4 \hat{O}_4 + \hat{O}_4 \hat{V}_4 - \hat{C}_4 \hat{C}_4 - \hat{S}_4 \hat{S}_4) (-\sqrt{q}) \times \right. \\
& \times \sum_{\epsilon=0,1} \sum_m \frac{q^{\frac{\alpha'}{2}(m+\frac{1}{\alpha'} B \epsilon)^T} g^{-1}(m+\frac{1}{\alpha'} B \epsilon) \gamma_\epsilon}{\hat{\eta}^4(-\sqrt{q})} + \\
& - 2^{-2} \sum_{i=1}^{2^{4-r}} I_D^i (\hat{V}_4 \hat{O}_4 + \hat{O}_4 \hat{V}_4 + \hat{C}_4 \hat{C}_4 + \hat{S}_4 \hat{S}_4) (-\sqrt{q}) \times \\
& \times \sum_{\epsilon=0,1} \sum_n \frac{q^{\frac{1}{2\alpha'} n^T g n} e^{\frac{2i\pi}{\alpha'} n^T B \epsilon} \tilde{\gamma}_\epsilon}{\hat{\eta}^4(-\sqrt{q})} + \\
& + I_N (\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 - \hat{C}_4 \hat{C}_4 + \hat{S}_4 \hat{S}_4) (-\sqrt{q}) \left( \frac{\hat{\vartheta}_3^2 \hat{\vartheta}_4^2}{\hat{\eta}^4} \right) (-\sqrt{q}) + \\
& \left. - \sum_{i=1}^{2^{4-r}} I_D^i (\hat{V}_4 \hat{O}_4 - \hat{O}_4 \hat{V}_4 + \hat{C}_4 \hat{C}_4 - \hat{S}_4 \hat{S}_4) (-\sqrt{q}) \left( \frac{\hat{\vartheta}_3^2 \hat{\vartheta}_4^2}{\hat{\eta}^4} \right) (-\sqrt{q}) \right\}
\end{aligned}$$

from the Möbius amplitude. As expected from our general considerations,  $\mathcal{M}$  involves the signs  $\gamma_\epsilon$  and  $\tilde{\gamma}_\epsilon$ , that play the same role as in the supersymmetric case.

As in [13], the cancellation of R-R tadpoles fixes both the overall size of the Chan-Paton gauge group, that in this case, however, is reduced by a factor  $2^{r/2}$  both on the D9 and D5-branes, and the group breaking pattern. However, the untwisted NS-NS scalars develop non-vanishing one-point functions, and thus a scalar potential arises.

Expanding the annulus and Möbius amplitudes to leading order in  $q$ , one can easily appreciate the role of the signs  $\gamma_\epsilon$  and  $\tilde{\gamma}_\epsilon$  that even in this case must coincide. One has

$$\begin{aligned}
\mathcal{A}_0^{(r)} = & \frac{1}{4} \left[ (I_N^2 + R_N^2 + I_D^2 + R_D^2)(V_4 O_4 - C_4 C_4) + (I_N^2 - R_N^2 + I_D^2 - R_D^2)(O_4 V_4 - S_4 S_4) + \right. \\
& \left. + 2^{r/2} (I_N I_D + R_N R_D)(V_4 C_4 - C_4 O_4) + 2^{r/2} (I_N I_D - R_N R_D)(O_4 S_4 - S_4 V_4) \right]
\end{aligned}$$

for the annulus amplitude, and

$$\begin{aligned}
\mathcal{M}_0^{(r)} = & -\frac{1}{4} \left[ (\gamma_0 + 1) I_N (\hat{V}_4 \hat{O}_4 - \hat{C}_4 \hat{C}_4) - (\tilde{\gamma}_0 + 1) I_D (\hat{V}_4 \hat{O}_4 + \hat{C}_4 \hat{C}_4) + \right. \\
& \left. + (\gamma_0 - 1) I_N (\hat{O}_4 \hat{V}_4 - \hat{S}_4 \hat{S}_4) - (\tilde{\gamma}_0 - 1) I_D (\hat{O}_4 \hat{V}_4 + \hat{S}_4 \hat{S}_4) \right]
\end{aligned}$$

for the Möbius amplitude. It is then evident that the choice  $\gamma_0 = \tilde{\gamma}_0 = +1$  corresponds to

$$G_{\text{CP}} = \text{SO}(2^{4-r/2}) \otimes \text{SO}(2^{4-r/2}) \Big|_{99} \otimes \text{Sp}(2^{4-r/2}) \otimes \text{Sp}(2^{4-r/2}) \Big|_{\bar{5}\bar{5}}$$



while the choice  $\gamma_0 = \tilde{\gamma}_0 = -1$  results in unitary gauge groups both for the D9 and D $\bar{5}$ -branes:

$$G_{\text{CP}} = \text{U}(2^{4-r/2})\Big|_{99} \otimes \text{U}(2^{4-r/2})\Big|_{\bar{5}\bar{5}}.$$

As pointed out in [13], the gaugini of the unitary group are not lifted in mass, since they are not affected by the Möbius projection. In the following tables we report the massless spectra for the two cases.

$r = 2,$	$n_T^c = 13,$	$n_H^c = 8,$	$G_{\text{CP}} = \text{SO}(8)_{99}^2 \otimes \text{Sp}(8)_{\bar{5}\bar{5}}^2$
$\phi :$	$4(8, 8; 1, 1) \oplus 4(1, 1; 8, 8) \oplus 4(8, 1; 1, 8) \oplus 4(1, 8; 8, 1)$		
$\psi_\alpha :$	$(28, 1; 1, 1) \oplus (1, 28; 1, 1) \oplus (1, 1; 28, 1) \oplus (1, 1; 1, 28) \oplus (8, 1; 8, 1) \oplus (1, 8; 1, 8)$		
$\bar{\psi}_{\dot{\alpha}} :$	$(8, 8; 1, 1) \oplus (1, 1; 8, 8)$		

**Table 4.** Massless spectrum for  $\gamma_0 = \tilde{\gamma}_0 = +1$ .  $\phi$ ,  $\psi_\alpha$  and  $\bar{\psi}_{\dot{\alpha}}$  denote scalar fields, left-handed spinors and right-handed spinors, respectively.

$r = 4,$	$n_T^c = 11,$	$n_H^c = 10,$	$G_{\text{CP}} = \text{SO}(4)_{99}^2 \otimes \text{Sp}(4)_{\bar{5}\bar{5}}^2$
$\phi :$	$4(4, 4; 1, 1) \oplus 4(1, 1; 4, 4) \oplus 8(4, 1; 1, 4) \oplus 8(1, 4; 4, 1)$		
$\psi_\alpha :$	$(6, 1; 1, 1) \oplus (1, 6; 1, 1) \oplus (1, 1; 6, 1) \oplus (1, 1; 1, 6) \oplus 2(4, 1; 4, 1) \oplus 2(1, 4; 1, 4)$		
$\bar{\psi}_{\dot{\alpha}} :$	$(4, 4; 1, 1) \oplus (1, 1; 4, 4)$		

**Table 5.** Massless spectrum for  $\gamma_0 = \tilde{\gamma}_0 = +1$ .  $\phi$ ,  $\psi_\alpha$  and  $\bar{\psi}_{\dot{\alpha}}$  denote scalar fields, left-handed spinors and right-handed spinors, respectively.

$r = 2,$	$n_T^c = 13,$	$n_H^c = 8,$	$G_{\text{CP}} = \text{U}(8)_{99}^2 \otimes \text{U}(8)_{\bar{5}\bar{5}}^2$
$\phi :$	$8(36; 1) \oplus 8(1; 28) \oplus 8(8, 8)$		
$\psi_\alpha :$	$(\text{Adj}; 1) \oplus (1; \text{Adj}) \oplus 2(8; 8)$		
$\bar{\psi}_{\dot{\alpha}} :$	$2(36; 1) \oplus 2(1; 36)$		

**Table 6.** Massless spectrum for  $\gamma_0 = \tilde{\gamma}_0 = -1$ .  $\phi$ ,  $\psi_\alpha$  and  $\bar{\psi}_{\dot{\alpha}}$  denote scalar fields, left-handed spinors and right-handed spinors, respectively.

$r = 4,$	$n_T^c = 11,$	$n_H^c = 10,$	$G_{\text{CP}} = \text{U}(4)_{99}^2 \otimes \text{U}(4)_{5\bar{5}}^2$
$\phi :$	$8(10; 1) \oplus 8(1; 6) \oplus 16(4, 4)$		
$\psi_\alpha :$	$(\text{Adj}; 1) \oplus (1; \text{Adj}) \oplus 4(4; 4)$		
$\bar{\psi}_{\dot{\alpha}} :$	$2(10; 1) \oplus 2(1; 10)$		

**Table 7.** Massless spectrum for  $\gamma_0 = \tilde{\gamma}_0 = -1$ .  $\phi$ ,  $\psi_\alpha$  and  $\bar{\psi}_{\dot{\alpha}}$  denote scalar fields, left-handed spinors and right-handed spinors, respectively.

Thus, in the open sector supersymmetry is broken explicitly, as a result of different representations assigned to the bosonic and fermionic degrees of freedom in the  $\bar{5}\bar{5}$  sector and from a flip of chirality of spinors in the mixed  $9\bar{5}$  sector.

One can show that tadpole conditions guarantee the cancellation of the irreducible gauge and gravitational anomalies in  $D = 6$ , while the residual anomaly polynomials

$$\begin{aligned} \mathcal{I}_8^{(r)} = & \frac{2^{r/2}}{64} \left( 2^{(2-r)/2} \text{tr} R^2 - \text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_3^2 - \text{tr} F_4^2 \right)^2 + \\ & - \frac{2^{r/2}}{64} \left( \text{tr} F_1^2 + \text{tr} F_2^2 - \text{tr} F_3^2 - \text{tr} F_4^2 \right)^2 + \\ & + \frac{4 + 2^{r/2}}{64} \left( \text{tr} F_1^2 - \text{tr} F_2^2 + \text{tr} F_3^2 - \text{tr} F_4^2 \right)^2 + \\ & + \frac{4 - 2^{r/2}}{64} \left( \text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_3^2 + \text{tr} F_4^2 \right)^2 \end{aligned}$$

and

$$\begin{aligned} \mathcal{I}_8'^{(r)} = & \frac{2^{r/2}}{16} \left( 2^{-r/2} \text{tr} R^2 - \text{tr} F_1^2 - \text{tr} F_2^2 \right)^2 + \frac{2^{r/2}}{16} \left( \text{tr} F_1^2 - \text{tr} F_2^2 \right)^2 + \\ & + \frac{2^{r/2}}{6} \text{tr} F_1 \left( -\frac{1}{16} \text{tr} R^2 \text{tr} F_2 + \text{tr} F_2^3 \right) + \frac{2^{r/2}}{6} \text{tr} F_2 \left( -\frac{1}{16} \text{tr} R^2 \text{tr} F_1 + \text{tr} F_1^3 \right) \end{aligned}$$

do not factorise and require a generalised Green-Schwarz mechanism [21].

## 7. Conclusions

In this paper we have investigated in some detail the open descendants of the  $T^4/Z_2$  and  $T^6/Z_3$  orbifolds in the presence of a non-vanishing background for the NS-NS antisymmetric tensor, thus generalising the constructions of [8] and of [3,2,9,10,12]. As expected,

the total size of the Chan-Paton gauge group is reduced according to the rank of  $B_{ab}$  both on the D9 and on the D5 branes. As already noticed in [11,7], the behaviour of the fixed points under the world-sheet parity  $\Omega$  depends on the antisymmetric tensor. As in [2], this yields varying numbers of tensor multiplets for the  $Z_2$  case. This is crucial for the correct interpretation of the amplitudes and is consistent with the constraints of two-dimensional conformal field theory in the presence of boundaries and/or crosscaps [4]. Moreover, as in [8], the Möbius amplitude involves sign ambiguities that are crucial for the correct interpretation of the amplitudes and, in the  $Z_2$  orbifold, allow one to connect continuously orthogonal and symplectic gauge groups. These signs are needed to enforce a correct normalisation of the Möbius amplitude, and at rational points they reduce to the discrete Wilson lines of [9], that affect the  $P$  transformation for  $\mathcal{M}$ . We have also shown how, in this case, couplings between twisted scalars and vector fields indeed arise, as demanded by the generalised Green-Schwarz mechanism [21]. It would be interesting to extend this construction to other six and four-dimensional orbifolds and to study the effect of discrete Wilson lines also in these cases.

We have also applied these results to a recently proposed type I scenario, where supersymmetry is broken on the branes but is unbroken in the bulk [13]. The outcome is, again, varying numbers of tensor multiplets and gauge groups of reduced rank. In this case, the signs in the Möbius amplitude allow a continuous deformation from products of orthogonal and symplectic groups to products of unitary groups.

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